Integrated Optimization Model for Industrial Manufacturing Operations Costs

Ezeaku I I^{1*} Okafor B², Obiukwu O², Onuoha O², Nwadinobi C P¹

¹Department of Mechanical Engineering, Abia State University, Uturu, Nigeria. ²Department of Mechanical Engineering, Federal University of Technology, Owerri, Nigeria. * Corresponding author's e-mail: <u>ezeakuinnocentmnse@gmail.com</u>. DOI: 10.56201/ijemt.vol.11.no6.2025.pg16.26

Abstract

Optimizing costs associated with manufacturing operations is a critical challenge undergone by manufacturing companies while navigating a fiercely competitive business environment. This research work has introduced application of MILP technique in manufacturing operations costs optimization and decision making. The research was conducted in a manufacturing firm in southern part of Nigeria that has two plants and twenty (20) locations for their products. Within the period under review, the first level/company's analytical technique which it has been adopting to reduce its manufacturing operations costs was found to be one billion, seven hundred and ninety eight million, six hundred and sixty thousand, seven hundred and thirty naira (\aleph 1798660730.00k). The developed Mixed Integer Linear programming (MILP) technique significantly reduced the company's manufacturing operations cost to one billion, six hundred and seventy eight million, one hundred thousand, two hundred and forty six naira (\aleph 1678100246.00k). The model showed manufacturing operations cost reduction of one hundred and twenty million, five hundred and sixty thousand, four hundred and eighty four naira (\aleph 120560484). This accounts for manufacturing operations cost reduction of the demand at various locations met from the plant. We believe that this result would be applicable to similar operations.

Keywords: Logistics operations, mixed-integral, cost optimization, inventory, decision making

I. Introduction

Manufacturing operations management involves managing a network of interconnected businesses, optimizing product flows, and reducing transportation costs [1]. It encompasses various activities in the distribution of final products to the end users [1]. The issue of manufacturing operations cost optimization is necessary for most manufacturing companies, such as paint, cement, oil and gas, and bottling, to mention a few [2].Cost optimization can be carried out at various stages of manufacturing: in-bound, in-process and out-bound manufacturing since manufacturing companies' response to customers' demand for products at various level of inbound and outbound operations [2]. The process involved is quite fuzzy or complex. Solving such complex problems in logistics operation or supply chain requires optimizing the production process and distribution operations. The issue of cost optimization is necessary for almost all - manufacturing companies, such as paint, cement, oil and gas, and bottling, to mention a few [2]. Designing an effective logistics network involves determining facility locations and capacities, sourcing demand, and selecting transportation modes to ensure cost-efficient customer service [3]. A lack of an adequately designed distribution network is a major problem for most

companies, especially those in developing countries [4]. As a result, their out-bound operations performance is as indicated by performance measurement perspectives [4]. In the manufacturing industry, an efficient network is vital for improving performance and meeting market demand consistently [5]. A standard out-bound manufacturing operation typically consists of manufacturers, main and local distribution centers (DCs), and destination zones such as manufacturing, pharmaceutical and hospitals where each level or echelon of the distribution may comprise numerous facilities. Most production managers believe that most distribution problems and inventory management cases can be solved using traditional approach which fails to embrace proper industrial manufacturing operations [6].

Optimization models form a significant area of research in various disciplines, including mathematics, operations research, engineering, and computer science. These models aim to find the best possible solution for a given problem by optimizing certain objective functions and subject to specific constraints [7]. One widely studied category of optimization models is linear programming (LP), which involves linear relationships between variables and constraints. Integer programming (IP) deals with optimization problems where the decision variables must take integer values. These models are often more challenging to solve compared to linear programming. Optimization models use mathematical algorithms and techniques to determine the best allocation of resources and optimize decision-making [8]. These models aim to find optimal solutions that minimize costs, maximize efficiency, or achieve other desired objectives. Optimization models can be used to address various cost optimization problems in manufacturing, including production planning, scheduling, capacity allocation, inventory management, and supply chain optimization [9]. Optimization models define an objective function that quantifies the goal to be achieved, such as minimizing costs or maximizing profit. It also has decision variables which represent the decision options available, such as production quantities, resource allocations, or inventory levels [10]. Optimization models incorporate constraints that reflect the limitations and requirements of the manufacturing system, such as capacity constraints, demand constraints, or resource availability constraints [11], embarked on an insightful journey by leveraging fuzzy set theory to encapsulate the intricate nature of uncertainties present within manufacturing operations dynamics. In pursuit of enhanced tactical out-bound manufacturing operations planning, they constructed a fuzzy linear programming model. This innovative model was tailored to address the complexities inherent in a multi-plant, multi-product, multi-level, and multi-period distribution network. [12] presented an integrated model that synchronized the inventory at retailers and warehouse, and the demand at the warehouse. One of the contributions of this paper was the development of an inventory-logistics framework that includes inventory and transportation components in the total cost function. [13] applied lagrangian relaxation (UI.R) to solve a MIP model for an integrated production distribution system. A heuristic was also adopted to assist in the problem solving which increases the overall efficiency of the given procedure. [14] came up with an integrated production distribution problem in multi-facility, multi-product and multi-period environment. The problem was approached as a network flow problem with an objective to match products with production lines to minimize the associated costs. [15] proposed memetic algorithm with population management (MA/PM) to solve integrated inventory distribution problem (IIDP) with one facility, one product and multiple customers with deterministic demand for multiple time-period set. The team's test results on problem instances with 20 time periods and 50, 100 and 200 customers showed that MA/PA outperforms the two given phase heuristic and greedy randomized adaptive search procedure (GRASP).

For many years, researchers have been involved only in one piece of the overall problem such as production or transportation sub-models which are being optimized separately and the solutions were then joined together to establish manufacturing operations policies and failed to take an integrated approach of an entire manufacturing operations. The scope of this research is centered on the development and application of Mixed Integer Linear Programming technique to optimize manufacturing operations costs. The study adopts a case study approach, focusing on a multi-plant, multi-destination paint manufacturing company to explore and address the complexities of cost management across key operational domains, including procurement, distribution, and logistics networks. This research aims to provide a comprehensive and integrated framework for cost optimization by leveraging MILP techniques. These methods are employed to model and solve the intricate challenges associated with manufacturing operations, ensuring efficient and cost-effective solutions. The objective is to bridge the theoretical frameworks with the practical nuances inherent in the case study company's manufacturing operations using Lingo Model (15.0 Version) as the primary tool for implementing and analyzing optimization models in the research methodology.

II. Materials and Methods

The tool for collecting information was a one-on-one interview asking the questions presented in the interview question section and use of questionnaire which were distributed and well explained to the participants.

In this research work, optimization of manufacturing operations costs in a multi-plant, multilocation outfit had been studied from a paint manufacturing industry in southern Nigeria by implementing Mixed Integer Linear Programming (MILP) technique aimed at exploring the potential integration technique to provide valuable insights into their application in manufacturing operations cost optimization in the developing economy like Nigeria where it has been under studied and aims to address the limitations of traditional methods, improve operational efficiency and provide manufacturing companies with robust, cost-effective and sustainable solutions.

Model Notation and Formulation

A mathematical model to assist decision making in an integrated production- inventorydistribution system can be developed. The model formulated will attempt to minimize total outbound manufacturing operations cost by simultaneously considering facility location, production schedule, inventory decision, distribution batch size and so on. The following notations were defined in other to model such a problem.

- i Index for plants, $i = 1, 2, \dots, I$
- j Index for locations, $j = 1, 2, \dots, J$
- k Index for plants, $i = 1, 2, \dots, k$
- i Index for products, $i = 1, 2, \dots, L$
- m Index for in-bound load-carriers, of $m = 1, 2, \dots, M$
- n Index for out-bound load-carriers, $n = 1, 2, \dots, N$
- i Index for time periods, t = 1, 2, ... T

Parameters and Decision Variables

- A_{jkn} Fixed production cost for product *l* at plant *i* in a given period
- B_{ijk} Variable cost for producing a unit of product *l* at plant *i* in a given period
- C_{ijk} Inventory cost for carrying a unit of product *l* at plant *i* in a given period

 D_{jki} Demand for product *l* by customer *k* in a given period

 E_{ijk} Inventory cost for carrying a unit of product *l* in Location *j* in a given period

 F_{ijk} Transportation cost for transporting a unit of product *l* from plant *I* to location *j* when using a carrier *m* in a given period

 P_{ij} Amount of product *l* produced at plant *i* in a given period

 P_{ijk} Driver capacity of out-bound load-carriers *m* in a given period

V_{*ij*} Inventory level of product l at plant I in a given period

 V_{ijk} Starting inventory level for product *l* at plant *i*

 W_{jki} Starting inventory level for product *l* in location *j*

 X_{ijk} Amount of product *l* transported from plant *i* to location *j* when using in-bound load-carriers say *m* in a given period

 Y_{ijk} Amount of product *l* transported from location *j* to customer *k* when using out-bound load-carrier say *m* in a given period

 Z_{jkn} Transportation requirement (break/consolidation bulk) of customer k for product l in a given period.

Objective Function

The objective function is to minimize the manufacturing operations cost Q, including fixed costs, variable production costs, inventory costs both at plants and locations and distribution costs.



Production capacity of the company

The two plants of the company, Portharcourt (WF1) and Yenegoa (WF2) are operating at about 82 percent of maximum production capacity which is used for practical purposes is shown in table 1 and periodic availability of products per plant from 2017 to 2021 in table 2.

		Years	1 0	•			
Plant	2017	2018	2019	2020	2021	Total	
WF1	3600000	3620000	3050000	4700000	5820000	20790000	
WF2	3420000	3680000	3705000	3350000	4540000	18695000	

Table 1: Maximum Production Capacity of the company per annum.

Transportation Rates

The distance between a specific source and a specific destination is a function of the cost of transportation of products. The warehouses at Portharcourt and Yenegoa are integrated with the plant. The average transportation cost per case-slot per kilometer is found to be $\frac{1}{2.55}$ in a round trip considering relevant carrier and operational costs.

Table 2: Periodic availability of products per plant from 2017-2021.

Years							
Plant	2017	2018	2019	2020	2021	Availability	
WF1	3600000	3620000	3050000	4700000	5820000	4158000	
WF2	3420000	3680000	3705000	3350000	4540000	3739000	

The Manufacturer's/ First level Technique

The manufacturer's approach to their manufacturing operations cost determination is an analytical (classical) mathematical technique. The total manufacturing operations cost obtained through this analytical technique was one billion, seven hundred and ninety eight million, six hundred and sixty thousand, seven hundred and thirty naira (H1798660730.00k).

Application of Mixed Integer Linear Programming (MILP) Model to Optimize Out-Bound Manufacturing Operations Costs.

Substituting the parameters into the general form of the model for the out-bound manufacturing operations cost problem developed in equations 1 to 5, we have

Objective function to minimize the out-bound manufacturing operations cost:

 $\begin{array}{l} \mbox{Minimize } Q = 6415110Z_{II} + 6515110Z_{I2} + 6615110Z_{I3} + 6615110Z_{I4} + 6615110Z_{I5} + 6698132Z_{21} \\ + 6798132Z_{22} + 6898132Z_{23} + 6808130Z_{24} + 6989132Z_{25} + 140.75P_{II} + 142.00P_{I2} + 160.42P_{I3} + \\ 173.45P_{I4} + 183.45P_{I5} + 140.50P_{21} + 142.75P_{22} + 160.42P_{23} + 173.45P_{24} + 183.45P_{25} + 11.50V_{II} + \\ 11.50V_{I2} + 12.50V_{I3} + 12.50V_{I4} + 15.00V_{I5} + 11.50V_{21} + 11.50V_{22} + 12.50V_{23} + 12.00V_{24} + \\ 15.50V_{25} + 102.51X_{II} + 39.78X_{I2} + 402.9X_{I3} + 0.00X_{I4} + 167.28X_{I5} + 217.50X_{I6} + 155.55X_{I7} + \\ 400.35X_{I8} + 271.06X_{I9} + 314.93X_{II0} + 396.78X_{II1} + 474.55X_{II2} + 450.33X_{II3} + 530.65X_{II4} + \\ 52.28X_{II5} + 522.24X_{II6} + 464.61X_{II7} + 414.61X_{II8} + 327.42X_{II9} + 524.02X_{I20} + 230.77X_{21} + \\ 536.26X_{22} + 79.81X_{23} + 477.10X_{24} + 402.90X_{25} + 539.32X_{26} + 535.50X_{27} + 400.35X_{28} + 544.68X_{29} \\ + 433.5X_{210} + 506.68X_{211} + 460.02X_{212} + 485.77_{213} + 513.06_{214} + 435.03X_{215} + 544.17_{216} + 0.00X_{217} \\ + 500.82X_{218} + 521.48X_{219} + 613.53X_{220} + 12.50W_{I} + 12.00W_{2} + 12.20W_{3} + 13.024 + 13.00W_{5} + \\ \end{array}$

 $13.00W_6 + 12.02W_7 + 12.20W_8 + 14.50W_9 + 13.50W_{I0} + 12.20W_{I1} + 12.40W_{I2} + 11.50W_{I3} + 12.00W_{I1} + 12.40W_{I2} + 11.50W_{I3} + 12.00W_{I1} + 12.00W_{I2} + 11.50W_{I3} + 12.00W_{I1} + 12.00W_{I2} + 11.50W_{I3} + 12.00W_{I1} + 12.00W_{I2} + 11.50W_{I3} + 12.00W_{I3} + 12.00W_{I1} + 12.00W_{I2} + 11.50W_{I3} + 12.00W_{I3} + 12.00W_{I3}$ $13.60W_{I4} + 14.20W_{I5} + 13.50W_{I6} + 12.60W_{I7} + 12.00W_{I8} + 13.50W_{I9} + 13.50W_{20} + 100.40Y_{IK} + 100.40W_{IK} + 100.40W_{IK$ $100.00Y_{2K} + 106.00Y_{3K} + 108.00Y_{4K} + 112.00Y_{5K} + 110.00Y_{6K} + 106.60Y_{7K} + 105.40Y_{8K} + 106.00Y_{6K} + 106.$ $104.00Y_{9K} + 106.00Y_{10K} + 110.00Y_{11K} + 106.00Y_{12K} + 102.00Y_{13K} + 109.20Y_{14K} + 105.00Y_{15K} + 100.00Y_{15K} + 100.00Y_{15K}$ $104.40Y_{16} + 104.60Y_{17K} + 106.30Y_{18K} +$ $100.80Y_{I9K} + 100.40Y_{20K}$ Subject to $P_{II} + P_{I2} + P_{I3} + P_{I4} + P_{I5}$ \leq 207900007 $P_{21} + P_{22} + P_{23} + P_{24} + P_{25}$ 186950008 \leq $V_{I1} + V_{I2} + V_{I3} + V_{I4} + V_{I5}$ \leq 22000009 \leq $V_{21} + V_{22} + V_{23} + V_{24} + V_{25}$ 200000010 $X_{I1} + X_{I2} + X_{I3} + X_{I4} + X_{I5} + X_{I6} + X_{I7} + X_{I8} + X_{I9} + X_{110} + X_{I11} + X_{I12} + X_{I13} + X_{I14}$ $+ X_{I15} + X_{I16} + X_{I17} + X_{I18} + X_{I19} + X_{I20} + X_{21} + X_{22} + X_{23}$ $+ X_{24} + X_{25} + X_{26} + X_{27} + X_{28} + X_{29}$ $+ X_{210} + X_{211} + X_{212} + X_{213} + X_{214}$ $+ X_{215} + X_{216} + X_{217} + X_{218}$ < 7065011 $+ X_{219} + X_{220}$ $Y_{IK} + Y_{2K} + Y_{3K} + Y_{4K} + Y_{5K} + Y_{6K} + Y_{7K} + Y_{8K} + Y_{9K} + Y_{10K}$

8222737012

 $+ Y_{11K} + Y_{12K} + Y_{13K} + Y_{14K} + Y_{15K} + Y_{16K}$

 $+ Y_{17K} + Y_{19K} + Y_{20K}$

\mathbf{W}_1	\leq	110000	 12a
W_2	\leq	110000	 12b
W_3	\leq	100000	 12c
W_4	\leq	100000	 12d
W_5	\leq	120000	 12e
W_6	\leq	120000	 12f
W_7	\leq	120000	 12g
W_8	\leq	120000	 12h
W9	\leq	100000	 12i
W_{10}	\leq	100000	 12j
W_{11}	\leq	120000	 12k
W_{12}	\leq	100000	 121
W13	\leq	120000	 12m
W_{14}	\leq	120000	 12n
W_{15}	\leq	120000	 120
W_{16}	\leq	100000	 12p
W_{17}	\leq	120000	 12q
W_{18}	\leq	120000	 12r
W19	\leq	100000	 12s
W_{20}	\leq	100000	 12t
Y_{1k}	\leq	120000	 13a
Y_{2k}	\leq	120000	 13b

 \leq

IIARD – International Institute of Academic Research and Development

Page 21

International Journal of Engineering and Modern Technology (IJEMT) E-ISSN 2504-8848 P-ISSN 2695-2149 Vol 11. No. 6 2025 www.iiardjournals.org online version

Y_{3k}	\leq	120000		13c		
Y_{4k}	\leq	120000		13d		
Y_{5k}	\leq	120000		13e		
Y _{6k}	\leq	120000		13f		
Y_{7k}	\leq	120000		13g		
Y_{8k}	\leq	120000		13h		
Y _{9k}	\leq	120000		13i		
Y_{10k}	\leq	120000		13j		
Y_{11k}	\leq	120000		13k		
Y_{12k}	\leq	120000		131		
Y _{13k}	\leq	120000		13m		
Y_{14k}	\leq	120000		13n		
Y _{15k}	\leq	120000		130		
Y _{16k}	\leq	120000		13p		
Y_{17k}	\leq	120000		13q		
Y _{18k}	\leq	120000		13r		
Y19k	\leq	120000		13s		
Y _{20k}	\leq	120000		13t		
$Y_{IK} + Y_{III} + Y_{III} + Y_{III}$ + Y_{III} Also, X_{11}, X Y_{1K} P_{11} V_{11}	$\begin{array}{llllllllllllllllllllllllllllllllllll$					
\mathbf{W}_{1}	••••	••••••••••••••••••••••	$W_{20} > 0$			
••1	• • • • •					

III. Results and Discussion

The graphical analysis of the periodic availability of products per plant from 2017 to 2021 is shown in figure 1. while the average transportation cost in Naira/case-slot between locations (warehouse) and customers is graphically illustrated in figure 2.



Figure 1: Periodic availability of products per plant from 2017-2021.



Figure 2: Average transportation cost in Naira/case-slot between plants (WF1 and WF2) and locations (warehouse).

A mixed integer linear programming model was formulated for the manufacturing operations cost problem of the organization as developed in equations 1 to 14. The developed model was solved with using LINGO programming software. The application reduced the manufacturing operations cost of the firm from one billion, seven hundred and ninety eight million, six hundred and sixty thousand, seven hundred and thirty naira (N1798660730.00k) to one billion, six hundred and seventy eight million, one hundred thousand, two hundred and forty six naira (N1678100246.00k) as shown in table3.

	Plants		Quantity	
Locations (depot)	Portharcourt	Yenegoa	Demand (case-slots)	
Omoku	4287500	-	4287500	
Oyibo	3741850	-	3741850	
Ahoada	-	4200380	4200380	
Portharcourt	4276510	-	4276510	
Owerri	-	4276510	3960910	
Umuahia	4160620	-	4160620	
Aba	4086130	-	4086130	
Calabar	-	4424940	4424940	
Eket	-	4143650	4143650	
Uyo	-	4300330	4300330	
Onisha	4320580	-	4320580	
Awka	-	4660380	4660380	
Omoba	4241850	-	4241850	
Asaba	-	3929680	3929680	
Onne	-	3712030	3712030	
Ekpoma	4294940	-	4294940	
Yenegoa	4295010	-	4295010	
Nnewi	-	3925000	3925000	
Ikotekpene	-	3324940	3324940	
Okigwe	3940140	-	3940140	
Prod.cap./case-slot 20790000 18695000				
Average Inventory Cost/year : N288300.00k				
Total Manufacturing Operations Cost: N1,678100246.00k				

Table 3: MILP output result summary of manufacturing operations cost optimization of the company

Cost reduction of \$120,560,484.00k or approximately 6.7% compared to the traditional analytical method was achieved. This result has demonstrated the capability of MILP to optimize manufacturing operations cost while meeting demand at various locations for customer satisfaction.

IV. CONCLUSION AND RECOMMENDATION

This research underscores the transformative potential of mixed integer linear programming (MILP) techniques in optimizing manufacturing operations costs. By addressing the research objectives and employing advanced methodologies, the study has demonstrated significant improvements in cost efficiency compared to traditional approaches as used by the industry. This model integrated key manufacturing operation activities, including production, inventory, and distribution, to provide a holistic approach to cost optimization. The research contributes to the growing body of knowledge on cost optimization and provides a practical framework for manufacturing companies to enhance their operational efficiency and competitiveness. However,

The MILP model developed could be adapted to other manufacturing companies for cost optimization.

Acknowledgement: we wish to thank the management staff of the industries that assisted us during this research work.

REFERENCES

- Tarimoradi, M., Zarandi, M.H., Zaman, H. and Turksan, I.B. (2017). Evolutionary fuzzy intelligent system for multi-objective supply chain network designs: An agent-based optimization state of the art. J. Intell. Manuf. 28, 1551–1579.
- Liu, Y., Lu, Y., Yu, L. and Wang, X. (2021). Cost optimization of material handling in a stochastic assembly line with carbon emission constraints. International Journal of Production Research, 59(3), 781-799. doi: 10.1080/00207543.2020.1802353.
- Diabat, A.; Dehghani, E.; Jabbarzadeh (2017). A. Incorporating location and inventory decisions into a supply chain design problem with uncertain demands and lead times. J. Manuf. Syst. 43, 139–149.
- Cameron, A., Ewen, M., Ross-Degnan, D., Ball, D. and Laing, R. (2009). Medicine prices, availability, and affordability in 36 developing and middle-income countries: A secondary analysis. *Lancet* 373, 240–249.
- Zahiri, B., Jula, P. and Tavakkoli-Moghaddam, R. (2018).Design of a pharmaceutical supply chain network under uncertainty considering perishability and substitutability of products. *Inf. Sci.* 423, 257–283.
- Uzorh, A., Nnanna, I. and Ezeaku, I. (2016). Determination of Work-in-Progress Inventory cost in a Multi-stage, Multi-product Manufacturing Process. The International Journal of Engineering and Science (IJES). 3:6, 20-28. ISSN(e)2519.
- Deng, G., and Feriss M. (2007,).A simulation-based logistics assessment framework in global pharmaceutical supply chain networks. J. Oper. Res. Soc. 74, 1242–1260.
- Wen H. and Rein S., 2010). A. Robust-fuzzy approach optimization in design of sustainable lean supply chain network under certainty. *Comput. Appl. Math.* 41, 255.
- Dhaenens-Flipo and Finke (2017). A new humanitarian relief logistic network for multi-objective optimization under stochastic programming. *Appl. Intell.* 52, 13729–1376.
- Pando, V., San-José, L.A., García-Laguna, J. and Sicilia, J. (2018). Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization. Comput. Ind. Eng. 117,81–93. [Cross Ref]
- Peidro F and Zupta N. (2017). Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain networks problems. Expert system with applications. 37,4488-4495. Hhtp://dx.doi.org/10.1016/J.eswa 2009.12.062.
- Ghasemi, P., Goodarzian, F. and Abraham, (2022). A new humanitarian relief logistic network for multi-objective optimization under stochastic programming. *Appl. Intel.* 52, 13729–1376.
- Gong, Q., Lai, K and Wang, S.(2008). Supply chain networks. Closed Jackson Network models and properties. International Journal of production economics.113.567-574.
- Li,B.,Wang X.,Tan G and Xiao G (2020). Ahybrid optimization approach for cost optimization of a multi-echelon closed-loop supply chain considering uncertainty demand and return. Journal of cleaner production.270,122361.
- Santoso T., Ahmed S., Goetschalckx M and Shapiro A (2016). A stochastic programming approach for supply chain network design under uncertainty.Eur. J. Oper. Res. 167,96-115